Let $f$ be the function whose graph is shown on the right. The zeros of $f$ are $-2, 1$ and $3$. **SCORE: ______ / 3 PTS**

Let $A = \int_{1}^{-2} f(x)\,dx$, $B = \int_{-2}^{3} f(x)\,dx$, $C = \int_{-2}^{1} f(x)\,dx$ and $D = \int_{1}^{1} f(x)\,dx$.

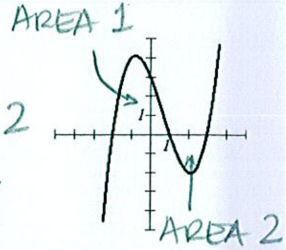
D = 0

C = AREA 1 > 0

B = AREA 1 − AREA 2

B > 0 BUT B < C

A = −B < 0




[a] Among the four quantities $A, B, C, D$, the greatest quantity is C. $1\frac{1}{2}$

[b] Among the four quantities $A, B, C, D$, the least quantity is A. $1\frac{1}{2}$

The speed of a runner at time $t$ is given by $v(t) = 4 + 2\cos t$ meters per second. **SCORE: ______ / 4 PTS**

Using left endpoints and $3$ subintervals, estimate the distance she ran from time $t = 0$ seconds to time $t = 2\pi$ seconds.

$\Delta t = \frac{2\pi - 0}{3} = \frac{2\pi}{3}$

$t_0 = 0$

$t_1 = 0 + \frac{2\pi}{3} = \frac{2\pi}{3}$

$t_2 = 0 + 2 \cdot \frac{2\pi}{3} = \frac{4\pi}{3}$

$v(t_0)\Delta t + v(t_1)\Delta t + v(t_2)\Delta t$

$= \underbrace{6}_{\frac{1}{2}} \cdot \frac{2\pi}{3} + \underbrace{3}_{\frac{1}{2}} \cdot \frac{2\pi}{3} + \underbrace{3}_{\frac{1}{2}} \cdot \frac{2\pi}{3}$ (1)

$= \underbrace{8\pi}_{1}$ $\underbrace{\text{METERS}}_{\frac{1}{2}}$

OK IF FACTORED OUT

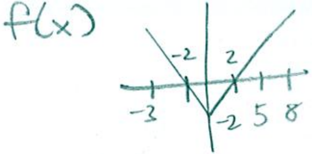
Let $f(x) = |x| - 2$. **SCORE: ______ / 8 PTS**

Evaluate the following integrals using the Properties of the Integral, and by interpreting in terms of area(s).

**<u>To earn full credit, you must show the use of the Properties of the Integral, NOT JUST ARITHMETIC, as part of your work.</u>**

**<u>NOTE: 0 points if you use the Fundamental Theorem of Calculus instead.</u>**

[a] $\int_{-3}^{5} f(x)\,dx$




$= \underbrace{\int_{-3}^{-2} f(x)\,dx + \int_{-2}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx}$ 2

$= \frac{1}{2}(1)(1) - \frac{1}{2}(2)(4) + \frac{1}{2}(3)(3)$

$= \underbrace{\frac{1}{2}} - \underbrace{4} + \underbrace{\frac{9}{2}}$ 1 1 1

$= \underbrace{1}$ 1

[b] $\int_{8}^{5} f(x)\,dx$

$= \underbrace{-\int_{5}^{8} f(x)\,dx}$ 1

$= -\frac{1}{2}(3)(3+6)$

$= \underbrace{-\frac{27}{2}}$ 1

☆ SEE ALTERNATE SOLUTIONS IF YOU SPLIT INTEGRAND INTO 2 FUNCTIONS

Let $f(x) = |x| - 2$. **SCORE: ______ / 8 PTS**

Evaluate the following integrals using the Properties of the Integral, and by interpreting in terms of area(s).

**To earn full credit, you must show the use of the Properties of the Integral, NOT JUST ARITHMETIC, as part of your work.**

**NOTE: 0 points if you use the Fundamental Theorem of Calculus instead.**

[a] $\int_{-3}^{5} f(x)\,dx$


$= \int_{-3}^{5} |x|\,dx - \int_{-3}^{5} 2\,dx$

$= \int_{-3}^{0} |x|\,dx + \int_{0}^{5} |x|\,dx - \int_{-3}^{5} 2\,dx$ 2

$= \frac{1}{2}(3)(3) + \frac{1}{2}(5)(5) - 2(5 - {}^{-}3)$

$= \frac{9}{2} + \frac{25}{2} - 16$ 1 1 1 1

$= 1$ 1

[b] $\int_{8}^{5} f(x)\,dx$

$= -\int_{5}^{8} f(x)\,dx$ 1

$= -\left(\int_{5}^{8} |x|\,dx - \int_{5}^{8} 2\,dx\right)$ $\frac{1}{2}$

$= -\left(\frac{1}{2}(3)(5+8) - 2(8-5)\right)$

$= -\left(\frac{39}{2} - 6\right)$

$= \frac{-27}{2}$ $\frac{1}{2}$

Write $\int_1^4 f(x)\,dx - \int_1^7 f(x)\,dx - \int_9^4 f(x)\,dx$ as a single integral. **SCORE: ______ / 3 PTS**

**To earn full credit, you must show the use of the Properties of the Integral, as part of your work.**

$$\int_1^4 f(x)\,dx \; \tfrac{1}{2} \; \underbrace{+\int_7^1 f(x)\,dx}\; \underbrace{+\int_4^9 f(x)\,dx} \; \tfrac{1}{2}$$

$$= \underbrace{\int_7^4 f(x)\,dx + \int_4^9 f(x)\,dx \quad \text{OR} \quad \int_7^1 f(x)\,dx + \int_1^9 f(x)\,dx} \quad \text{EITHER ONE OK}$$

1

$$= \underbrace{\int_7^9 f(x)\,dx} \; 1$$

**In complete sentences, using proper English and mathematical notation,** state the definition of the definite integral given in the website handout.

SCORE: ______ / 3 PTS

SEE HANDOUT ON MY WEBSITE

Using the form of the definition of the definite integral given in Theorem 4 (ie. right hand sum), **SCORE: ______ / 9 PTS**

evaluate $\int_1^4 (4x-6)\,dx$. **NOTE: 0 points if you use geometry or the Fundamental Theorem of Calculus instead.**

REMEMBER THIS OVERRIDING RULE

$\Delta x = \frac{4-1}{n} = \frac{3}{n}$  $x_i = 1 + i\left(\frac{3}{n}\right) = 1 + \frac{3i}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f\left(1+\frac{3i}{n}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(4\left(1+\frac{3i}{n}\right)-6\right)\frac{3}{n} \quad 1 \quad 1 \quad 1$$

$$= \lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n}\left(-2+\frac{12i}{n}\right) \quad 1$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\sum_{i=1}^{n} -2 + \frac{12}{n}\sum_{i=1}^{n} i\right) \quad 1$$

$$= \lim_{n\to\infty} \frac{3}{\cancel{n}}\left(-2\cancel{n} + \frac{\overset{6}{\cancel{12}}}{n}\frac{n(n+1)}{\cancel{2}}\right) \quad \tfrac{1}{2} \quad 1$$

$$= \lim_{n\to\infty} 3\left(-2+\frac{6(n+1)}{n}\right) \quad 1$$

$$= 3(-2+6) \quad \tfrac{1}{2}$$

$$= 12 \quad \tfrac{1}{2}$$

$+\frac{1}{2}$ IF YOU WROTE "$\lim_{n\to\infty}$" ON EACH LINE SHOWN